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AN ENTIRE FUNCTION SHARING A LINEAR POLYNOMIAL WITH ITS LINEAR DIFFERENTIAL POLYNOMIALS.

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ABSTRACT. In this paper we study the uniqueness of an entire function when it shares a linear polynomial with its linear differential polynomials.

1. INTRODUCTION, DEFINITIONS AND RESULTS

Let f be a noncostant meromorphic function defined in the open complex plane $\mathbb{C}$. The integrated counting function of poles of f is defined by

$$N(r,\infty;f) = \int_0^r \frac{n(t,\infty;f) - n(0,\infty;f)}{t}dt + n(0,\infty;f)\log r.$$

where $n(t,\infty;f)$ be the number of poles of f lying in $|z| \le r$, the poles are counted according to their multiplicities and $n(0,\infty;f)$ be the multiplicity of pole of f at origin.

For a polynomial $a = a(z)$, $N(r,a;f)\,(\overline{N}(r,a;f))$ be the integrated counting function (reduced counting function) of zeros of $f-a$ in $|z| \le r$.

Let $A \subset \mathbb{C}$, we denote by $n_A(r,a;f)$ the number of zeros of $f-a$, counted with multiplicities, that lie in $\{z : |z| \le r\} \cap A$. The corresponding integrated counting function $N_A(r,a;f)$ is defined by

$$N_A(r,a;f) = \int_0^r \frac{n_A(t,a;f) - n_A(0,a;f)}{t}dt + n_A(0,a;f)\log r.$$

We also denote by $\overline{N}_A(r,a;f)$ the reduced counting functions of those zeros of $f-a$ that lie in $\{z : |z| \le r\} \cap A$.

Clearly if $A = \mathbb{C}$, then $N_A(r,a;f) = N(r,a;f)$ and $\overline{N}_A(r,a;f) = \overline{N}(r,a;f)$.

We denote by $E(a,f)$ the set of zeros of $f-a$ counted with multiplicities and by $\overline{E}(a,f)$ the set of distinct zeros of $f-a$.

For the standard definitions and notations of the value distribution theory authors suggest to see [1] and [8]

The investigation of uniqueness of an entire function sharing certain values with its derivatives was initiated by L. A. Rubel and C. C. Yang [7] in 1977. They proved the following result.

Theorem A. [7]. *Let f be a non-constant entire function. If $E(a;f) = E(a;f^{(1)})$ and $E(b;f) = E(b;f^{(1)})$, for distinct finite complex numbers a and b, then $f \equiv f^{(1)}$.*

In 1979 E.Mues and N.Steinmetz [6] took up the case of IM sharing in the place of CM sharing of values and proved the following theorem.

Theorem B. [6]. *Let f be a non-constant entire function and a, b be two distinct finite complex values and . If $\overline{E}(a;f) = \overline{E}(a;f^{(1)})$ and $\overline{E}(b;f) = \overline{E}(b;f^{(1)})$, then $f \equiv f^{(1)}$.*

The uniqueness of an entire function sharing a nonzero finite value with its first two derivatives was considered by G. Jank, E. Mues and L. Volkmann [2] in 1986. The following is their result.

Theorem C. [2]. *Let f be a nonconstant entire function and a be a nonzero finite value. If $\overline{E}(a;f) = \overline{E}(a;f^{(1)}) \subset \overline{E}(a;f^{(2)})$, then $f \equiv f^{(1)}$.*

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